

1. Aditya and Pico live 1000 meters apart, and they start walking toward each other's houses at the same time. Pico walks at a constant speed of 65 meters per minute. Aditya walks at a constant speed of 60 meters per minute. How many minutes after they start walking will they meet? (Do NOT include any units in your answer.)

Answer: 8

Solution: The distance between Aditya and Pico decreases by $65 + 60 = 125$ meters per minute. Therefore, they meet after

$$\frac{1000}{125} = \boxed{8}$$

minutes.

2. Let a and b be real numbers such that $a + b = 16$ and $a^3 - b^3 = ab^2 - a^2b + 2048$. Compute ab^2 .

Answer: 192

Solution: Rearranging gives $a^3 + a^2b - ab^2 - b^3 = 2048$. The left-hand side can be factored as $(a + b)(a^2 - b^2) = (a + b)^2(a - b)$. Therefore, $(16^2)(a - b) = 2048$, and $a - b = 8$. Thus, $a = 12$ and $b = 4$, so $ab^2 = 12 \cdot (4)^2 = 12 \cdot 16 = \boxed{192}$.

3. A trading card store only sells small and large packs of cards, which cost \$11 and \$18, respectively. Each small pack has the same number of cards, each large pack has the same number of cards, and a large pack has more cards than a small pack. The greatest number of cards that can be bought with \$38 is 48, and the greatest number of cards that can be bought with \$56 is 75. What is the greatest number of cards that can be bought with \$202? In each scenario, not all the money needs to be spent.

Answer: 273

Solution: Let a small pack contain a cards and a large pack contain b cards. Then $a < b$.

With \$38, one could purchase 3 small packs, 1 small pack plus 1 large pack, or 2 large packs. So the number of cards is $\max(3a, a + b, 2b) = \max(3a, 2b) = 48$.

With \$56, one could purchase 5 small packs, 3 small packs plus 1 large pack, 1 small pack plus 2 large packs, or 3 large packs. So the number of cards is $\max(5a, 3a + b, a + 2b, 3b) = \max(5a, 3a + b, 3b) = 75$.

We can further simplify: if $5a \geq 3b$, then $a \geq \frac{3}{5}b$, so

$$5a = 3a + 2a \geq 3a + 2 \cdot \frac{3}{5}b = 3a + \frac{6}{5}b \geq 3a + b.$$

If $5a \leq 3b$, then $b \geq \frac{5}{3}a$, so

$$3b = 2b + b \geq 2 \cdot \frac{5}{3}a + b = \frac{10}{3}a + b \geq 3a + b.$$

Therefore, $\max(5a, 3a + b, 3b) = \max(5a, 3b) = 75$.

There are three cases to consider:

- If $a \leq \frac{3}{5}b$, then $\max(3a, 2b) = 2b = 48$ and $\max(5a, 3b) = 3b = 75$. But then $b = 24$ and $b = 25$, contradiction.

- If $a \geq \frac{2}{3}b$, then $\max(3a, 2b) = 3a = 48$ and $\max(5a, 3b) = 5a = 75$. But then $a = 16$ and $a = 15$, contradiction.
- If $\frac{3}{5}b \leq a \leq \frac{2}{3}b$, then $\max(3a, 2b) = 2b = 48$ and $\max(5a, 3b) = 5a = 75$. This gives $b = 24$ and $a = 15$.

Now, to find the greatest number of cards that can be acquired with \$202, we can note that $11 \times 18 = 198$ is close, so we can compare 18 small packs to 11 large packs. We have $18 \times 15 = 270$ and $11 \times 24 = 264$, so 18 small packs are better. So we start with 18 small packs, and we consider exchanges to utilize the extra \$4. The best exchange is to switch 3 small packs to 2 large packs, increasing the amount spent by \$3 for a net gain of $2 \times 24 - 3 \times 15 = 3$ cards; the other exchanges (such as 8 small packs to 5 large packs) do nothing at best. Thus, the greatest number of cards that can be acquired with \$202 is $270 + 3 = \boxed{273}$.