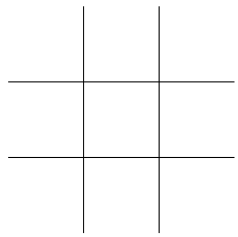
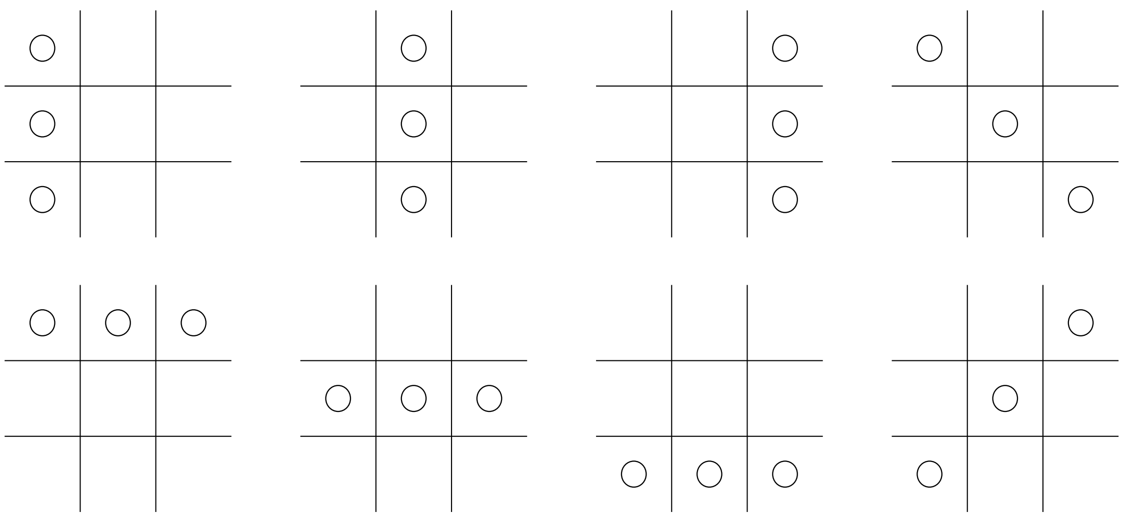
1. How many ways are there to place 3 identical $\bigcirc$ on a 3×3 tic-tac-toe board according to the following rules?

   - Each $\bigcirc$ is in a different cell.
   - Not all three $\bigcirc$ are in the same row, column, or diagonal.



   Answer: 76

   Solution: There are eight ways to place 3 $\bigcirc$s on the board such that they form a line of three.



   Since the three $\bigcirc$s are identical, there are $\frac{9 \cdot 8 \cdot 7}{6} = 84$ different ways to place 3 $\bigcirc$s on the board. This means there are $84 - 8 = \boxed{76}$ potential configurations.

2. Find the sum of all three-digit positive integers N such that N plus the reverse of N is a perfect cube. For example, the reverses of 245 and 370 are 542 and $073 = 73$, respectively.

   Answer: 663

   Solution: Let $N = 100A + 10B + C$ for digits A, B, and C. N plus its reverse is $101(A+C) + 20B$. List the perfect cubes that are obtainable by summing two three-digit numbers:

$$125, 216, 343, 512, 729, 1000, 1331, \text{ and } 1728.$$

   Note that $A + C$ has to have the same remainder as the cube when divided by 20. Thus, $A + C = 5, 16, 3, 12, 9, 0, 11, 8$ respectively.

   Since $20B$ adds at most 1 to the hundreds place ($B = 9$ yields $20B = 180$, and $A + C < 20$ in all cases), the digit in the hundreds place must be equal to d or $d + 1$, where d is the digit in the units place. Of the available cubes, only 343 (1000 doesn't work since $A+C=0$ is not allowed) satisfies this condition, so only $A+C=3$ yields any solutions.

We can then compute $20B = 343 - 101(3) \implies B = 2$, giving $N = 320, 221, 122$ as the only possible values of N. The requested sum is $320 + 221 + 122 = \boxed{663}$.

3. A subset T of $S = \{1, 2, \dots, 2025\}$ is *generational* if for every integer $1 \le j \le 2025$, there exist integers $c_1, c_2, \dots, c_{2025}$ such that

$$\sum_{n \in T} nc_n \equiv j \pmod{2025}.$$

What is the size of the largest subset T that is **not** generational?

Answer: 675

Solution: Each integer x that is coprime to 2025 has a multiplicative inverse c_x mod 2025; that is, there exists c_x such that $xc_x \equiv 1 \pmod{2025}$. If an integer coprime to 2025 is included in a subset, that subset is automatically generational: for all integers $1 \le j \le 2025$ we can write $j(xc_x) \equiv j \pmod{2025}$.

Furthermore, by Bezout's identity, for any two integers x, y such that $\gcd(x, y) = 1$, there exist integers c_x, c_y such that $xc_x + yc_y = 1$. Thus, any subset with two coprime integers is generational: for any integer $1 \le j \le 2025$ we can write

$$jxc_x + jyc_y = j(xc_x + yc_y) = j \equiv j \pmod{2025}.$$

If a subset $T = \{t_1, \dots, t_n\}$ is not generational, it must contain only numbers that are not pairwise coprime and that are not coprime to 2025.

We must have $\gcd(t_1, \dots, t_n, 2025) = d > 1$. Thus, any non-generational subset consists of multiples of d, where d is a factor of 2025. We can maximize the size of such a set by choosing $d > 1$ as small as possible and including every multiple of d from d up to 2025 itself.

The smallest factor of 2025 greater than 1 is 3, so the largest non-generational subset has $\boxed{675}$ elements:

$$1 \cdot 3, 2 \cdot 3, \dots, 675 \cdot 3 = 2025.$$