1. Aditya and Pico live 1000 meters apart, and they start walking toward each other's houses at the same time. Pico walks at a constant speed of 65 meters per minute. Aditya walks at a constant speed of 60 meters per minute. How many minutes after they start walking will they meet? (Do NOT include any units in your answer.)

   Answer: 8

   Solution: The distance between Aditya and Pico decreases by $65+60 = 125$ meters per minute. Therefore, they meet after
   $$\frac{1000}{125} = \boxed{8}$$
   minutes.

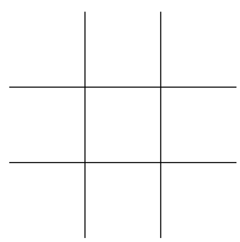
2. The minute hand of a circular clock reaches from the center of the clock to the edge, and the hour hand is half as long as the minute hand. The circumference of the clock is 24π. What is the area of the triangle formed by the hour hand and minute hand at 4:30 AM?

   Answer: $\mathbf{18\sqrt{2}}$

   Solution: Since the circumference is 24π, the radius is 12, so the minute hand has length 12 and the hour hand has length 6. The angle at 4:30 AM is $2(30) - \frac{1}{2}(30) = 45°$, so the area of the triangle is $\frac{1}{2}(12)(6)\sin(45°) = \boxed{18\sqrt{2}}$.

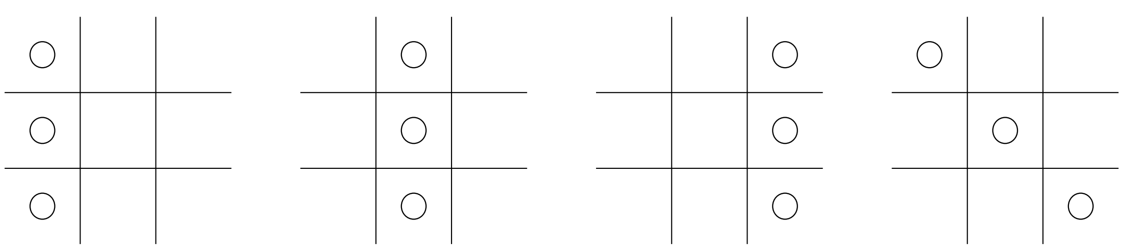
3. How many ways are there to place 3 identical $\bigcirc$ on a 3×3 tic-tac-toe board according to the following rules?

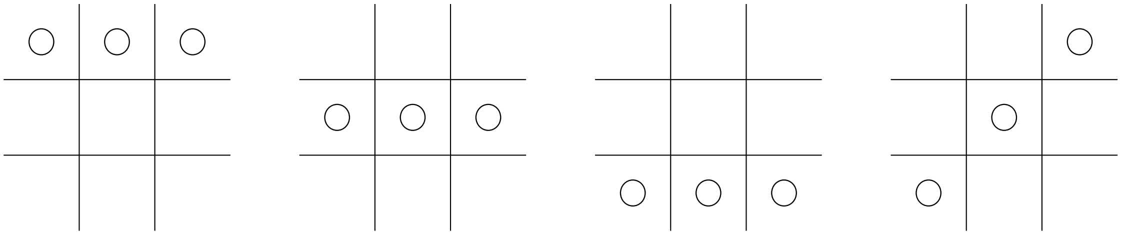
   - Each $\bigcirc$ is in a different cell.
   - Not all three $\bigcirc$ are in the same row, column, or diagonal.



   Answer: 76

   Solution: There are eight ways to place 3 $\bigcirc$s on the board such that they form a line of three.





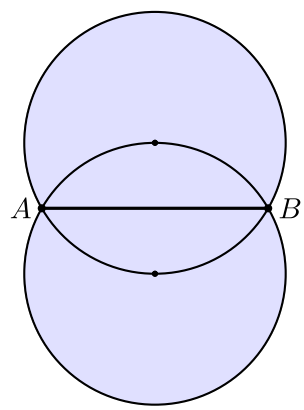
Since the three ◯s are identical, there are $\frac{9 \cdot 8 \cdot 7}{6} = 84$ different ways to place 3 ◯s on the board. This means there are $84 - 8 = \boxed{76}$ potential configurations.

4. Points A and B lie in the plane so that $AB = 1$. Compute the area of the region of the plane consisting of all points C such that $\angle ACB \geq 60°$.

   Answer: $\frac{4}{9}\pi + \frac{1}{2\sqrt{3}}$ OR $\frac{4}{9}\pi + \frac{\sqrt{3}}{6}$ OR $\frac{8\pi+3\sqrt{3}}{18}$

   Solution: For a fixed segment AB of length 1, the set of points C satisfying $\angle ACB = 60°$ consists of two circular arcs passing through A and B, one on each side of $\overline{AB}$. These arcs lie on the circumcircles of the two equilateral triangles having AB as a side.

   The circles each have radius $\frac{1}{\sqrt{3}}$, and their centers are $\frac{1}{\sqrt{3}}$ apart:



   The area of the union of the two disks is

$$A = 2 \cdot \frac{2\pi}{3} r^2 + 1 \cdot \frac{1}{2\sqrt{3}} = \boxed{\frac{4\pi}{9} + \frac{1}{2\sqrt{3}}}.$$

5. Find the sum of all three-digit positive integers N such that N plus the reverse of N is a perfect cube. For example, the reverses of 245 and 370 are 542 and $073 = 73$, respectively.

   Answer: 663

   Solution: Let $N = 100A+10B+C$ for digits A, B, and C. N plus its reverse is $101(A+C)+20B$. List the perfect cubes that are obtainable by summing two three-digit numbers:

$$125, 216, 343, 512, 729, 1000, 1331, \text{ and } 1728.$$

   Note that $A + C$ has to have the same remainder as the cube when divided by 20. Thus, $A + C = 5, 16, 3, 12, 9, 0, 11, 8$ respectively.

Since $20B$ adds at most 1 to the hundreds place ($B = 9$ yields $20B = 180$, and $A + C < 20$ in all cases), the digit in the hundreds place must be equal to d or $d + 1$, where d is the digit in the units place. Of the available cubes, only 343 (1000 doesn't work since $A + C = 0$ is not allowed) satisfies this condition, so only $A + C = 3$ yields any solutions.

We can then compute $20B = 343 - 101(3) \implies B = 2$, giving $N = 320, 221, 122$ as the only possible values of N. The requested sum is $320 + 221 + 122 = \boxed{663}$.