1. The minute hand of a circular clock reaches from the center of the clock to the edge, and the hour hand is half as long as the minute hand. The circumference of the clock is 24π. What is the area of the triangle formed by the hour hand and minute hand at 4:30 AM?

   Answer: $\mathbf{18\sqrt{2}}$

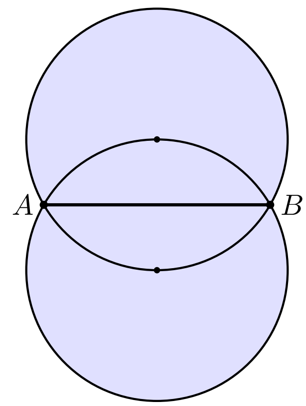
   Solution: Since the circumference is 24π, the radius is 12, so the minute hand has length 12 and the hour hand has length 6. The angle at 4:30 AM is $2(30) - \frac{1}{2}(30) = 45°$, so the area of the triangle is $\frac{1}{2}(12)(6)\sin(45°) = \boxed{18\sqrt{2}}$.

2. Points A and B lie in the plane so that $AB = 1$. Compute the area of the region of the plane consisting of all points C such that $\angle ACB \geq 60°$.

   Answer: $\mathbf{\frac{4}{9}\pi + \frac{1}{2\sqrt{3}}}$ **OR** $\mathbf{\frac{4}{9}\pi + \frac{\sqrt{3}}{6}}$ **OR** $\mathbf{\frac{8\pi+3\sqrt{3}}{18}}$

   Solution: For a fixed segment AB of length 1, the set of points C satisfying $\angle ACB = 60°$ consists of two circular arcs passing through A and B, one on each side of $\overline{AB}$. These arcs lie on the circumcircles of the two equilateral triangles having AB as a side.

   The circles each have radius $\frac{1}{\sqrt{3}}$, and their centers are $\frac{1}{\sqrt{3}}$ apart:



   The area of the union of the two disks is

$$A = 2 \cdot \frac{2\pi}{3} r^2 + 1 \cdot \frac{1}{2\sqrt{3}} = \boxed{\frac{4\pi}{9} + \frac{1}{2\sqrt{3}}}.$$

3. In triangle $\triangle ABC$ with circumcircle ω, point M is the midpoint of side $\overline{BC}$ and $AB < AC$. Line $\overleftrightarrow{AM}$ intersects ω at $D \neq A$. Let X and Y be points on ω such that $\overline{BX}$ and $\overline{CY}$ bisect $\overline{AM}$. Given $AM = 40$, $MD = 10$, and $BX = 50$, find CY.

   Answer: $\mathbf{30\sqrt{3}}$

   Solution:

   Let Z be the midpoint of AM, which is where BX and CY bisect AM. We have $AZ = ZM = 20$. Power of a point at M with respect to ω gives $BM \cdot MC = AM \cdot MD = 40 \cdot 10 = 400$. Thus, $BM = MC = 20$.

   Applying power of a point at Z with respect to ω gives $BZ \cdot ZX = AZ \cdot ZD = 20 \cdot 30 = 600$. Since $BX = 50$, it follows that BZ and ZX are 20 and 30 in some order. $AB < AC$ implies $\angle AMB = \angle ZMB$ is acute, therefore $BZ < 20\sqrt{2} < 30$. Thus, $BZ = 20$ and $ZX = 30$.

Now, note that $MB = MZ = MC = 20$, so M is the circumcenter of $\triangle BZC$. Angle $\angle BZC$ subtends diameter BC of the circumcircle of $\triangle BZC$, so it is a right angle. The Pythagorean theorem on $\triangle BZC$ gives $ZC = 20\sqrt{3}$. By power of a point at Z with respect to ω, $CZ \cdot ZY = 600 \implies YZ = \frac{600}{20\sqrt{3}} = 10\sqrt{3}$. Finally, $CY = CZ + ZY = \boxed{30\sqrt{3}}$.